TABLE II

RELATIVE POWER IN PROPAGATING AND EVANESCENT MODES[a]

No. of Mode	Reflected Power PROPAGATING	Reflected Power REACTIVE	Transmitted Power PROPAGATING	Transmitted Power REACTIVE
M= 1	PM/P INPUT= 0.003516	+J 0.0	PM/P OUTPUT= 0.691403	+J 0.0
M= 2	PM/P INPUT= 0.0	+J-0.080699	PM/P OUTPUT= 0.305081	+J 0.0
M= 3	PM/P INPUT= 0.0	+J 0.010024	PM/P OUTPUT= 0.0	+J 0.070690
M= 4	PM/P INPUT= 0.0	+J-0.009721	PM/P OUTPUT= 0.0	+J-0.045942
M= 5	PM/P INPUT= 0.0	+J 0.003003	PM/P OUTPUT= 0.0	+J 0.018121
M= 6	PM/P INPUT= 0.0	+J-0.003332	PM/P OUTPUT= 0.0	+J-0.009543
M= 7	PM/P INPUT= 0.0	+J 0.001491	PM/P OUTPUT= 0.0	+J 0.001279
M= 8	PM/P INPUT= 0.0	+J-0.001792	PM/P OUTPUT= 0.0	+J-0.000572
M= 9	PM/P INPUT= 0.0	+J 0.000925	PM/P OUTPUT= 0.0	+J 0.002257
M=10	PM/P INPUT= 0.0	+J-0.001161	PM/P OUTPUT= 0.0	+J-0.003982
M=11	PM/P INPUT= 0.0	+J 0.000667	PM/P OUTPUT= 0.0	+J 0.003604
M=12	PM/P INPUT= 0.0	+J-0.000363	PM/P OUTPUT= 0.0	+J-0.003352
M=13	PM/P INPUT= 0.0	+J 0.000547	PM/P OUTPUT= 0.0	+J 0.001207
M=14	PM/P INPUT= 0.0	+J-0.000738	PM/P OUTPUT= 0.0	+J-0.000402
M=15	PM/P INPUT= 0.0	+J 0.001301	PM/P OUTPUT= 0.0	+J 0.000073
M=16	PM/P INPUT= 0.0	+J-0.001295	PM/P OUTPUT= 0.0	+J-0.000492
M=17	PM/P INPUT= 0.0	+J 0.000658	PM/P OUTPUT= 0.0	+J 0.000926
M=18	PM/P INPUT= 0.0	+J-0.000767	PM/P OUTPUT= 0.0	+J-0.001510
M=19	PM/P INPUT= 0.0	+J 0.000419	PM/P OUTPUT= 0.0	+J 0.000543
M=20	PM/P INPUT= 0.0	+J-0.000517	PM/P OUTPUT= 0.0	+J-0.000815

[a] *Note:* $b/a = 1.4$, $f \cdot a = 15$ GHz·cm.

considered are presented in Table I for a frequency where TE_{11} and TM_{11} modes both propagate in the output guide. Mode power conversion and launch phase coefficients converge rapidly and are essentially constant after six to eight evanescent modes on each side of the discontinuity are included. The ratio of the number of input to output waveguide modes considered is not significant if more than six evanescent modes are included; this supports a similar conclusion by Clarricoats [6].

The relative reactive power in the evanescent modes is shown in Table II, in which 20 modes are considered. The modes are numbered in terms of increasing cutoff frequencies. The reactive power quickly diminishes as the mode order increases and the inductive or capacitive nature of the discontinuity is provided by the total reactive power.

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Radiation Loss from Open-Circuited Dielectric Resonators

J. WATKINS

***Abstract*—The Q factor of an open-circuited resonator is influenced by dielectric, conductor, and radiation losses. This short paper discusses these losses and shows that insight into the radiation loss can be obtained by an extension to the analysis given by Lewin. This shows that the radiation loss is a maximum for the dominant mode and that provided the permittivity is not less than 9.0 the radiation losses are at a minimum for the second mode.**

It is suggested that these results may be applied to the design of filters based on dielectric resonators. The application of the results to stripline resonators in which the dielectric extends past the termination of the upper conducting strip is more difficult, and it is suggested that experimental work is required to investigate postulated improvements. Finally, some of the radiation patterns of the open-circuited dielectric resonators, obtained in this paper, show interesting directional properties which may be applied to the design of antenna systems.

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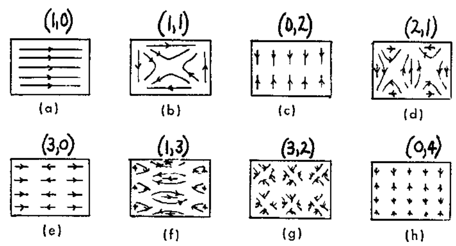

Fig. 1. Patterns of the conduction current for various modes.

I. INTRODUCTION

Resonators involving ceramic substrates whose upper and lower surfaces have been coated with metallic conductors have previously been reported by Napoli and Hughes [1]. To avoid the degenerate resonances which would occur with a square resonator, a rectangular geometry is chosen and simple theory shows that

$$\mathcal{f} = \frac{c}{2L_z} \frac{1}{\sqrt{\epsilon_r}} \left(m^2 + \frac{n^2}{\gamma^2} \right)^{1/2} \tag{1}$$

where

- $\mathcal{f}$ resonant frequency;
- c velocity of light;
- ϵ_r relative permittivity;
- L_z larger side;
- γL_z shorter side;
- γ <1;
- m, n integers.

The assumption is made here that there are no variations in the fields across the thickness of the dielectric. Fig. 1(a)–(h) shows some of the patterns of the conduction current for resonances with different combinations of m and n. Fig. 2 shows the dimensions of the resonator and the coordinate system used in this paper.

The Q factor of these resonances are due to losses from four possible mechanisms: 1) coupling with the load and generator; 2)

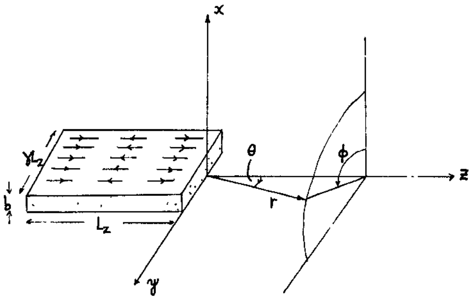

Fig. 2. Dimensions of the resonator and coordinate system used.

dielectric loss; 3) conductor loss; and 4) radiation loss. Assuming that the couplings can be reduced to produce negligible loading, then

$$\frac{1}{Q_{\text{resultant}}} = \frac{1}{Q_{\text{dielectric}}} + \frac{1}{Q_{\text{conductor}}} + \frac{1}{Q_{\text{radiation}}} \tag{2}$$

where

$Q_{\text{dielectric}}$ Q of the resonator if the losses due to conduction and radiation were both zero;
$Q_{\text{conductor}}$ similarly defined with respect to dielectric effects and radiation;
$Q_{\text{radiation}}$ similarly defined with respect to dielectric and conduction effects;
$Q_{\text{resultant}}$ overall Q resulting from the presence of all three effects.

Since the loss tangents for the high quality aluminas used in these substrates lie in the range $1\text{–}5 \times 10^{-4}$, then an upper limit for $1/Q_{\text{dielectric}}$ is

$$\frac{1}{Q_{\text{dielectric}}} = 0.0005. \tag{3}$$

In the case where no field variation occurs in the x-direction, the value for $1/Q_{\text{conductor}}$ may be obtained from the expression given by Watkins [2]

$$Q_{\text{conductor}} = \sqrt{4\pi f \mu_0 \sigma} \cdot b \tag{4}$$

where

σ conductivity;
μ_0 $4\pi 10^{-7}$.

Taking the following values for the parameters involved

$f = 6$ GHz
$\sigma = 5 \times 10^7$ S (for copper)
$b = 0.64$ mm (a commercially available substrate)

then

$$\frac{1}{Q_{\text{conductor}}} = 0.00142. \tag{5}$$

This short paper is concerned with the radiation loss which can be determined by an extension of Lewin's analysis of radiation from discontinuities in stripline [3]. The analysis which follows relates to modes of oscillation in which m or n equals zero, i.e., those in which the current flow is parallel to one edge of the resonator, and is uniform over the width perpendicular to this edge.

II. Analysis

The result for the Hertzian vector Π obtained for an infinite stripline is

$$\Pi = -\frac{60jb}{k}\,\frac{e^{-jkr}}{r}\int_{-\infty}^{\infty}\left(i_x\left[\frac{\epsilon_r - 1}{\epsilon_r}\right]\frac{\partial I}{\partial \zeta} - i_z \cdot jk \sin\theta\cos\phi \cdot I\right) e^{jk\zeta\cos\theta}\, d\zeta \tag{6}$$

where

i_x, i_z x and z unit vectors;
ϵ_r relative permittivity;
k $=2\pi/\lambda$;
λ free-space wavelength;
I strip current;
ζ current coordinate in the z-direction.

Among a number of cases that Lewin considered was that of an open-circuited stripline coinciding with $z=0$ and extending to infinity along the negative z axis, so that the appropriate limits of integration are 0 and $-\infty$. By assuming that the contribution of exponentials with imaginary arguments is zero at negative infinity he obtained an expression for Π. Using this he obtained expressions for the far-field components of the electric field, viz., E_θ and E_ϕ. By integrating $|E_\theta|^2 + |E_\phi|^2$ over a hemisphere he found the total power radiated to be

$$P = 60(kb)^2 \cdot F_1(\epsilon_r) \tag{7}$$

where

$$F_1(\epsilon_r) = \left(\frac{\epsilon_r + 1}{\epsilon_r}\right) - \frac{(\epsilon_r - 1)^2}{2\epsilon_r\sqrt{\epsilon_r}} \cdot \log_e\left(\frac{\sqrt{\epsilon_r} + 1}{\sqrt{\epsilon_r} - 1}\right). \tag{8}$$

In the case of the open-circuited resonator we may describe, with Lewin, the strip current I by a sinelike function

$$I = e^{-jk'\zeta} - e^{jk'\zeta} \tag{9}$$

where $k' = \sqrt{\epsilon_r}k$; but we replace the lower limit of the integral in (6) by $-n(\pi/k')$. (The nature of the excitation which gives rise to the presence of the current I and which can be realized by a number of different experimental techniques is not specified.) Here n, which takes integer values other than zero, represents the mode number. The upper limit of the integral remains, as before, equal to zero.

After some algebraic development the θ, ϕ components of the far electric fields become

$$E_\theta = -\frac{je^{-jkr}}{r} \cdot \frac{120(kb)}{\sqrt{\epsilon_r}} \cdot \cos\phi\left[1 - (-1)^n e^{(jn\pi\cos\theta/\sqrt{\epsilon_r})}\right] \tag{10}$$

$$E_\phi = \frac{je^{-jkr}}{r} \cdot \frac{120(kb)}{\sqrt{\epsilon_r}} \cdot \frac{(\epsilon_r - 1)}{(\epsilon_r - \cos^2\theta)} \cdot \cos\theta\sin\phi\left[1 - (-1)^n e^{(jn\pi\cos\theta/\sqrt{\epsilon_r})}\right] \tag{11}$$

which may be compared with Lewin's results for the semi-infinite open-circuited line, viz.,

$$E_\theta = -\frac{je^{-jkr}}{r} \cdot \frac{120(kb)}{\sqrt{\epsilon_r}} \cos\phi \tag{12}$$

$$E_\phi = \frac{je^{-jkr}}{r} \cdot \frac{120(kb)}{\sqrt{\epsilon_r}} \frac{(\epsilon_r - 1)}{(\epsilon_r - \cos^2\theta)} \cos\theta\sin\phi. \tag{13}$$

The influence of the modifying term

$$\left[1 - (-1)^n e^{(jn\pi\cos\theta/\sqrt{\epsilon_r})}\right]$$

leads to a value for the radiated power given by the expression

$$P(n, \epsilon_r) = 60(kb)^2 F_1'(n, \epsilon_r) \tag{14}$$

where

$$F_1'(n, \epsilon_r) = \int_0^\pi \frac{1}{\epsilon_r}\left(1 + \frac{\cos^2\theta[\epsilon_r - 1]^2}{[\epsilon_r - \cos^2\theta]^2}\right)\left(\left[1 - (-1)^n \cos\left\{\frac{n\pi\cos\theta}{\sqrt{\epsilon_r}}\right\}\right]^2 + \sin^2\left\{\frac{n\pi\cos\theta}{\sqrt{\epsilon_r}}\right\}\right)\sin\theta\, d\theta \tag{15}$$

which may be compared with Lewin's result

$$P(\epsilon_r) = 60(kb)^2\left[\left(\frac{\epsilon_r + 1}{\epsilon_r}\right) - \frac{(\epsilon_r - 1)^2}{2\epsilon_r\sqrt{\epsilon_r}}\log_e\left(\frac{\sqrt{\epsilon_r} + 1}{\sqrt{\epsilon_r} - 1}\right)\right]. \tag{16}$$

The integral in (15) has been evaluated by numerical methods using trapezoidal integration (details of an associated computer program are available on application to the author). Fig. 3(a)–(d) shows values of F_1' as a function of mode number for the permittivity values 3.8, 9.0, 30.0, and 80.0. These correspond to quartz, alumina, materials with zero temperature coefficients of permittivity, and lastly, titania. These results show a number of interesting features and are as follows.

1) As $n \to \infty$, $F_1' \to 2F$, i.e., twice the value obtained by Lewin for the single open-circuited end of a semi-infinite stripline. It therefore justifies his assumption that the contribution of exponentials with imaginary arguments at minus infinite is zero.

2) $F_1'(n, \epsilon_r)$ has a maximum value at $n=1$ and is nearly twice the value obtained by merely doubling F. This finding cor-

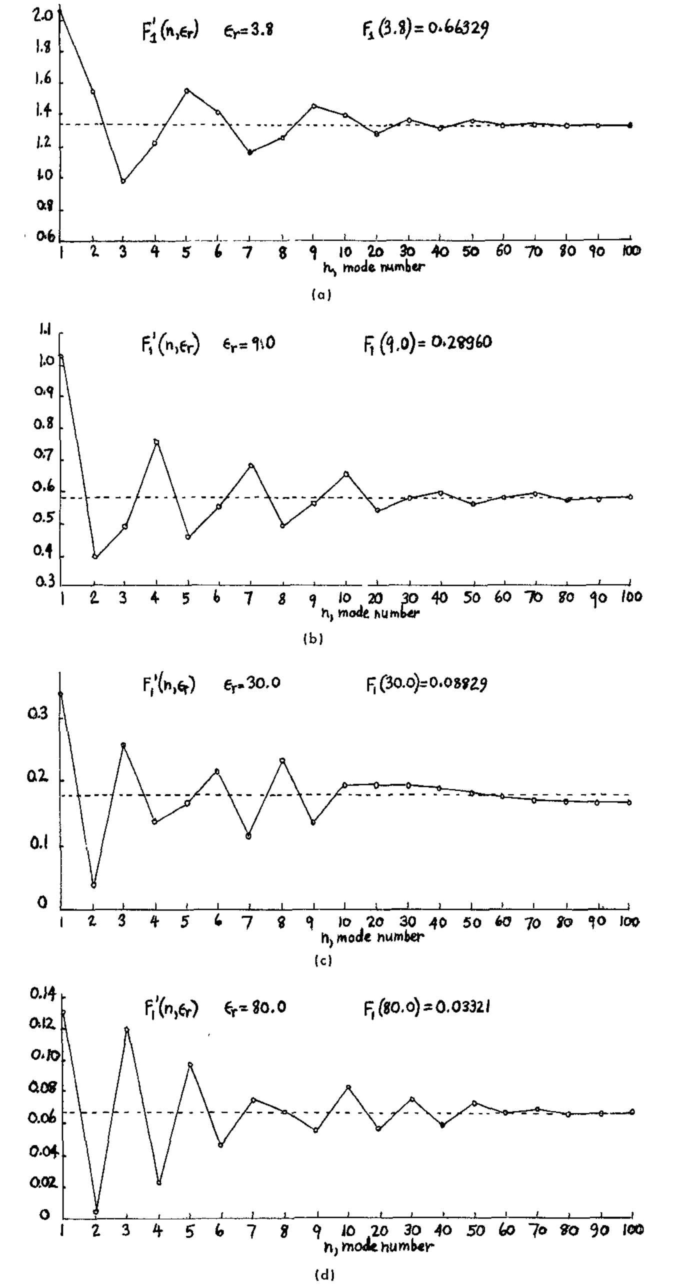

Fig. 3. $F_1'(n, \epsilon_r)$ as a function of mode number, n for $\epsilon_r = 3.8$, 9.0, 30.0, and 80.0.

roborates that of Easter and Roberts [4] obtained by an alternative analysis.

3) For $\epsilon_r \geq 9.0$, $F_1'(n, \epsilon_r)$ has a minimum value at $n = 2$.

With the above information we may now proceed to calculate $1/Q_{\text{rad}}$. Its expression is given as

$$\frac{1}{Q_{\text{rad}}} = \frac{P(n, \epsilon_r)}{2\pi \mathcal{f} L_{\text{res}}} \tag{17}$$

where L_{res} is the effective inductance of the resonator.

Since, using transmission line theory,

$$L_{\text{res}} = \frac{\mu_0 b n \lambda}{2W\sqrt{\epsilon_r}} \tag{18}$$

where W is the length of the side of the resonator perpendicular to the current flow (equal to L_z or γL_z of Fig. 2, depending on the mode $m0$ or $n0$, respectively), then

$$\frac{1}{Q_{\text{rad}}} = \frac{2W\sqrt{\epsilon_r}}{\mu_0 b n \lambda W} 60(kb)^2 F_1'(n, \epsilon_r). \tag{19}$$

Putting $\mathcal{f}$ in GHz and W, b in mm, then

$$\frac{1}{Q_{\text{rad}}} = \frac{1}{45\,000} \frac{W b \mathcal{f}^2 \sqrt{\epsilon_r}}{n} F_1'(n, \epsilon_r). \tag{20}$$

In the case where

$\mathcal{f} = 6$ GHz
$W = 25.4$ mm
$b = 0.64$ mm
$n = 1$
$\epsilon_r = 9.0$
$F_1' = 1.03896$

then

$$\frac{1}{Q_{\text{rad}}} = 0.0405. \tag{21}$$

Consequently, the overall Q for the three mechanisms for the mode $n = 1$ and the above values is given as

$$\frac{1}{Q_{\text{resultant}}} = \underbrace{0.0005}_{\text{dielectric loss}} + \underbrace{0.00142}_{\text{conductor loss}} + \underbrace{0.0405}_{\text{radiation loss}} \tag{22}$$

and it is seen that the radiation loss forms the major contribution. If however the mode $n = 2$ is used, then the expression for $1/Q_{\text{resultant}}$ becomes

$$\frac{1}{Q_{\text{resultant}}} = 0.0005 + 0.00142 + 0.0078 \tag{23}$$

and the two Q's are

$$\begin{gathered} n = 1 \\ Q_{\text{resultant}} = 23.6 \\ n = 2 \\ Q_{\text{resultant}} = 102.9. \end{gathered} \tag{24}$$

The radiated field can be conveniently studied by the E_ϕ component, i.e., the field vertical to the ground plane when $\phi = (\pi/2)$. Fig. 4(a)–(n) shows the radiation patterns for the various combinations of $n = 1(1)6$ and $\epsilon_r = 3.8$, 9.0, 30.0, and 80.0. In these figures the arrow denotes the direction represented by $\theta = 0$. The amplitudes of the electric field have been normalized to a common scale. Although the majority of the patterns resemble the "figure of eight" shape obtained by Lewin for the open-circuited semi-infinite stripline, a number are distinctively different and have four main lobes. It is difficult to forward a physical explanation of this phenomenon. Its origin would seem to be associated with the complicated nature of the modifying term appearing in (10) and (11).

III. Discussion

The results obtained above are significant in three distinct areas of application, viz., filters based on open-circuited dielectric resonators, stripline resonators, and antennas.

In the open-circuited dielectric resonator, (20) shows that provided F_1' is bounded, Q_{rad} increases with increasing mode number. The nature of the increase is especially interesting as shown by the results in the following table:

n	1	2	3	4	5
$n/F_1'(n, 3.8)$	0.481	1.31	3.03	3.31	3.22
$n/F_1'(n, 9.0)$	0.963	5.00	6.08	5.20	10.85
$n/F_1'(n, 30.0)$	2.93	46.2	11.35	28.6	29.8
$n/F_1'(n, 80.0)$	7.59	313.0	25.2	169.0	50.8

An increase in mode number means an increase in length, and in many applications, particularly in microelectronic engineering, design specifications may impose a limitation on available space. In this context, for ϵ_r values not less than 9.0, the mode $n = 2$ is a case to be noted for the design of filters based on open-circuited resonators, e.g., bandpass or bandstop filters using multiple resonators with staggered resonant frequencies. The attraction of the use of the mode $n = 2$ increases as the permittivity increases.

The application of the results obtained in this short paper to

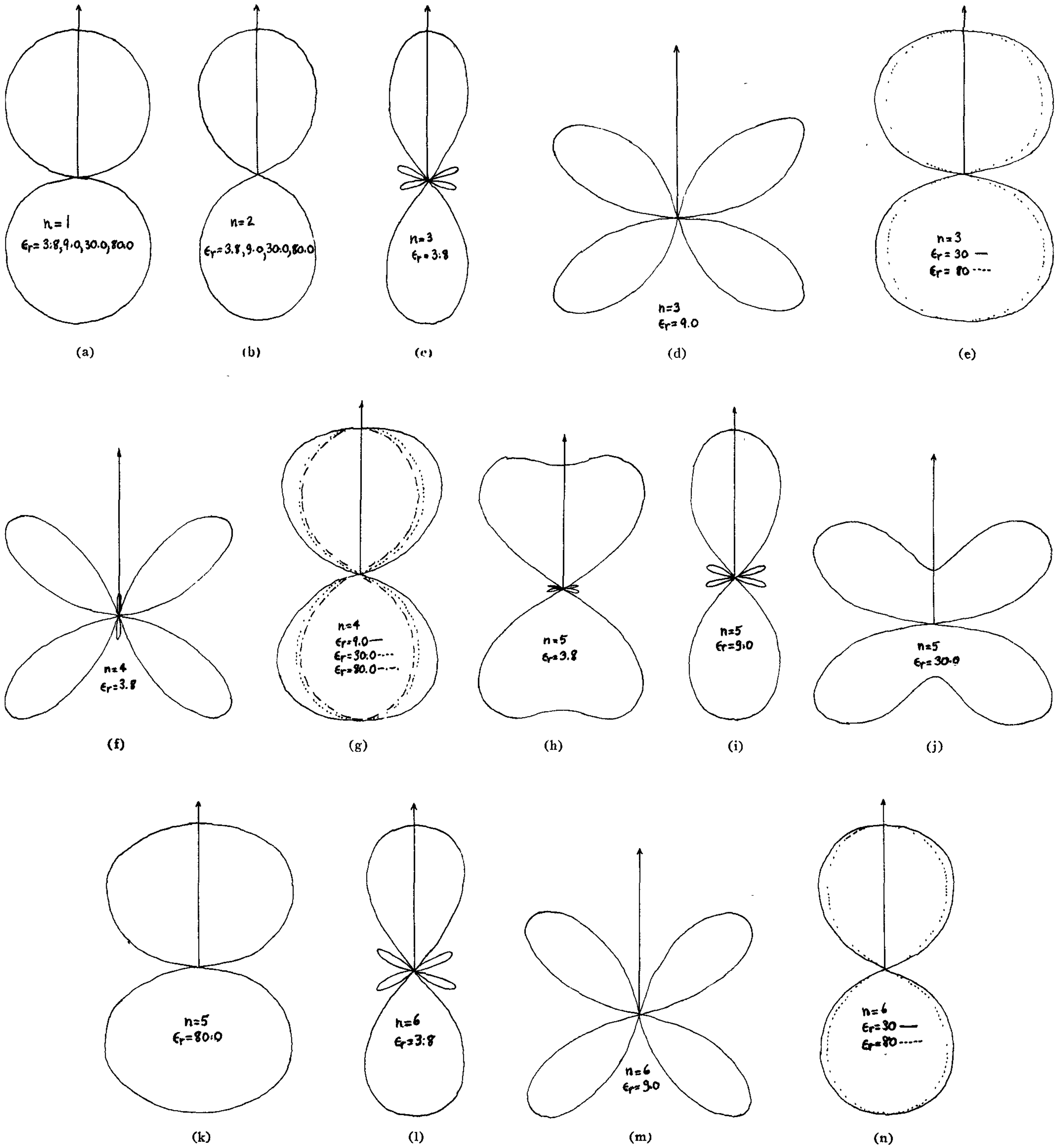

Fig. 4. Electric field patterns of E_ϕ versus θ for various combinations of mode number n and relative permittivity ϵ_r.

stripline resonators is more difficult to assess. In such resonators, the dielectric extends past the termination of the upper conducting strip and an analysis of the loss of radiation at the open-circuit is more difficult and may possibly be examined by Wiener–Hopf methods. In the absence of such an analysis it would now be useful to investigate the behavior of radiation loss as a function of mode number in such stripline resonators by experimental methods to see if the pattern in the table is maintained.

Lastly, the results of Fig. 4(a)–(n) have application to the field of antenna design. With a proper choice of mode number n, and relative permittivity ϵ_r, the directional qualities of an antenna can be realized; and for a particular n, ϵ_r combination, control of the parameters W and b allows a specified radiation impedance to be met.

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